A Unified Finite Strain Gradient-Enhanced Micropolar Continuum Approach for Modeling Quasi-Brittle Failure of Cohesive-Frictional Materials

Matthias Neuner[1], Richard A. Regueiro[2] & Christian Linder[1]

[1] Stanford University, Department of Civil & Environmental Engg. [2] University of Colorado Boulder, Department of Civil, Environmental & Architectural Engg.

Background

Cohesive-frictional materials like concrete, mortar, rock, tough ceramics, but also granular materials like soils, sands or powders constitute an essential part of many engineering and scientific applications. The behavior of these materials, governed by *friction* and *cohesion* between particles or grains, is characterized by a highly nonlinear, and common characteristics are

- inelastic behavior that is observed even at low stress levels,
- low resistance in tension compared to a potentially high compressive strength,
- increasing material strength with increasing stress confinement, and
- complex failure mechanisms.

In particular, in confined compression for many cohesive-frictional materials a rather ductile strain hardening behavior is followed by ductile strain softening, often accompanied by diffuse failure zones or highly localized zones of large inelastic deformations manifested by shear bands or fault zones, depending on the stress state. In contrast, certain cohesive-frictional materials like sands or soils do not sustain tensile stresses, whereas for other materials which exhibit cohesive bonds between particles, e.g., concrete or cemented granular materials, comparatively low tensile stresses lead to failure by cracking.

Accordingly, modeling the constitutive behavior of cohesive-frictional materials is a challenging task, and in this regard classical continuum models are often not sufficient. For standard models, zones of localized large strains potentially arise due to softening material behavior, but also due to non-associated plastic flow. In models for cohesive-frictional materials based on the theory of plasticity, non-associated plastic flow rules are commonly required for representing the volumetric inelastic behavior in a realistic manner. From a numerical point of view, deficiencies related to material failure and localized inelastic deformations are encountered due to the loss of ellipticity of the underlying initial-boundary value problem for the static case, or the loss of hyperbolicity for dynamic problems, resulting in pathological mesh dependence in finite element simulations.

Quasi-brittle cracking of cohesive-frictional materials

For many cohesive-frictional materials with distinct cohesive particle bonds, softening material behavior in tension or unconfined compression results in discrete cracks. For such materials, the formation of cracks is characterized by a fracture process zone of finite size ahead of the crack tip, in which microcracks emerge and coalesce to a distinct stress free macroscropic crack (Figure 1) – a process which is commonly denoted as quasi-brittle cracking.

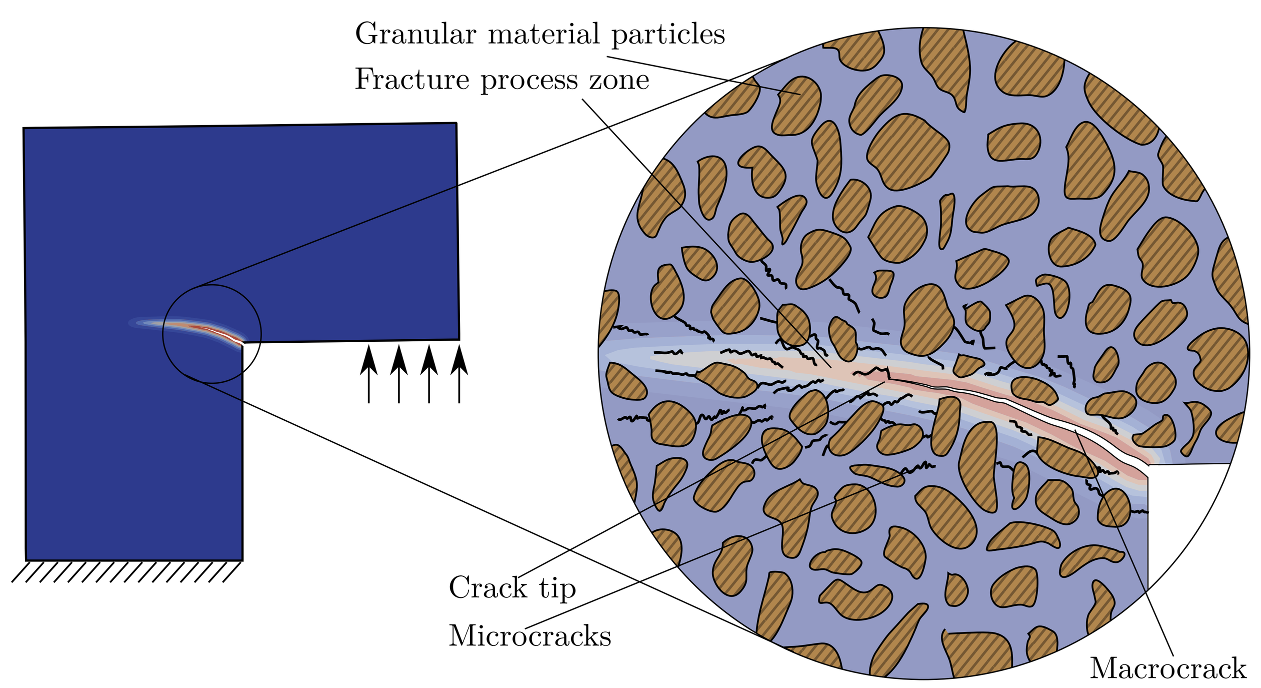

Figure 1: Failure in tension of cohesive-frictional materials is often observed in form of quasi-brittle cracking, characterized by a failure process zone of finite size in which microcracks coalesce to a distinct macroscopic crack.

Shear band formation in cohesive-frictional materials

In contrast to the material behavior in tension, in confined compression and shear for many cohesive-frictional materials highly localized zones of large inelastic deformations are observed, commonly in form of shear bands or fault zones. Similar to quasi-brittle fracture zones, for such localized zones of inelastic deformations the size effect of cohesive-frictional materials becomes apparent, which is for instance manifested by the finite width of shear bands. Clearly, the characteristic dimensions of such localized zones are related to deformations of the microstructure of a material, for instance in form of material particle rotations, depending on the scale of the microstructure or material heterogeneities.

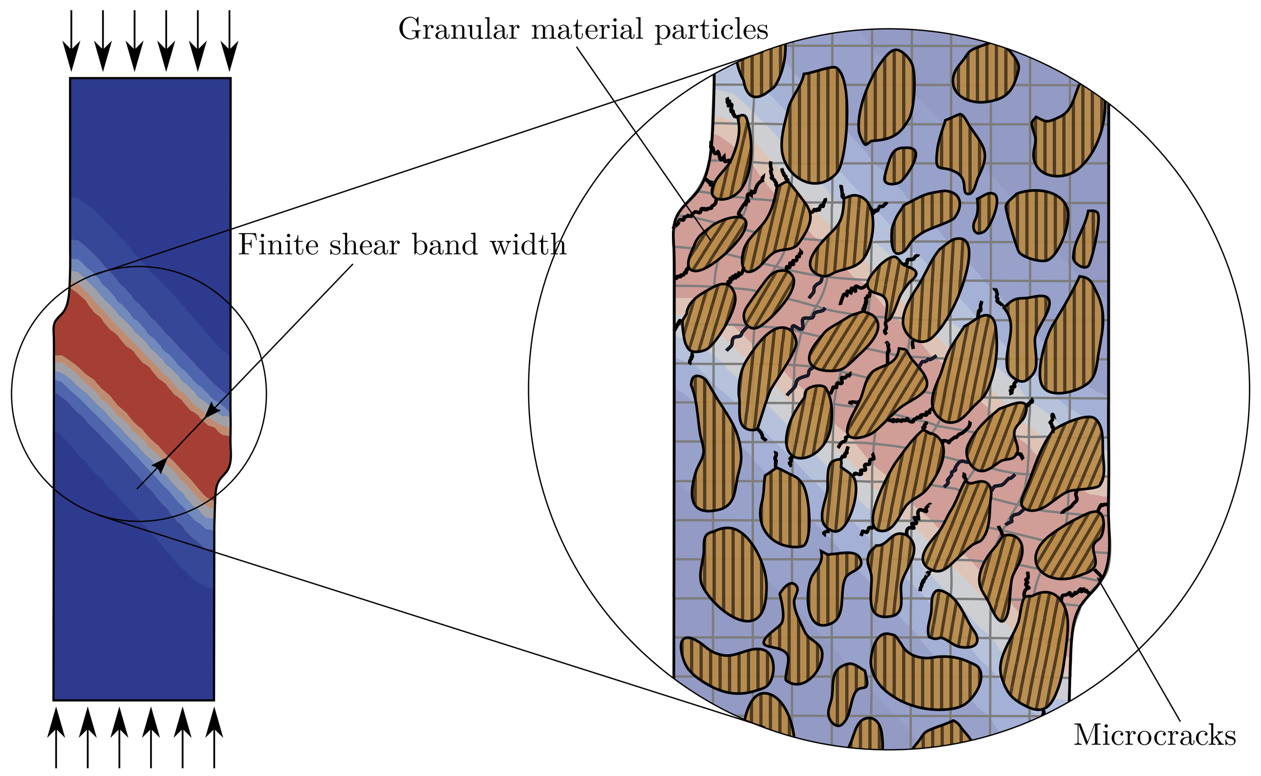

Figure 2: Shear bands in a cohesive-frictional material are characterized by a finite width, which is influenced by the deformation of microstructure of the material, e.g., the rotation of granular material particles. The rotation of these material particles is in general different from the macroscopically observed rotation of the structure.

Aim

For a general continuum model for cohesive-frictional materials, able to represent material failure under a broad range of loading conditions, both the size effect related to microstructural deformations and the nonlocal character of quasi-brittle failure in terms of interacting microcracks have to be taken in account. This is the focus of the present work:

It aims at a unified framework for modeling the mechanical behavior of cohesive-frictional materials, accounting for quasi-brittle failure and microstructural deformations, formulated in a geometrically exact, three-dimensional setting.

Major objectives:

- The proposal of a general continuum framework for constitutive models for cohesive-frictional materials, based on the combination of the gradient-enhanced continuum and the micropolar continuum, and formulated in a geometrically exact, three-dimensional setting.
- The realization of the proposed novel framework by constitutive models for particular cohesive-frictional materials in terms of numerically robust and efficient implementations into a finite element framework.
- The validation of the derived finite element implementation based on benchmark examples and experimental results for different cohesive-frictional materials, and a comparison with alternative approaches for modeling failure of cohesive-frictional materials in numerical simulations presented in the literature.

Continuum framework

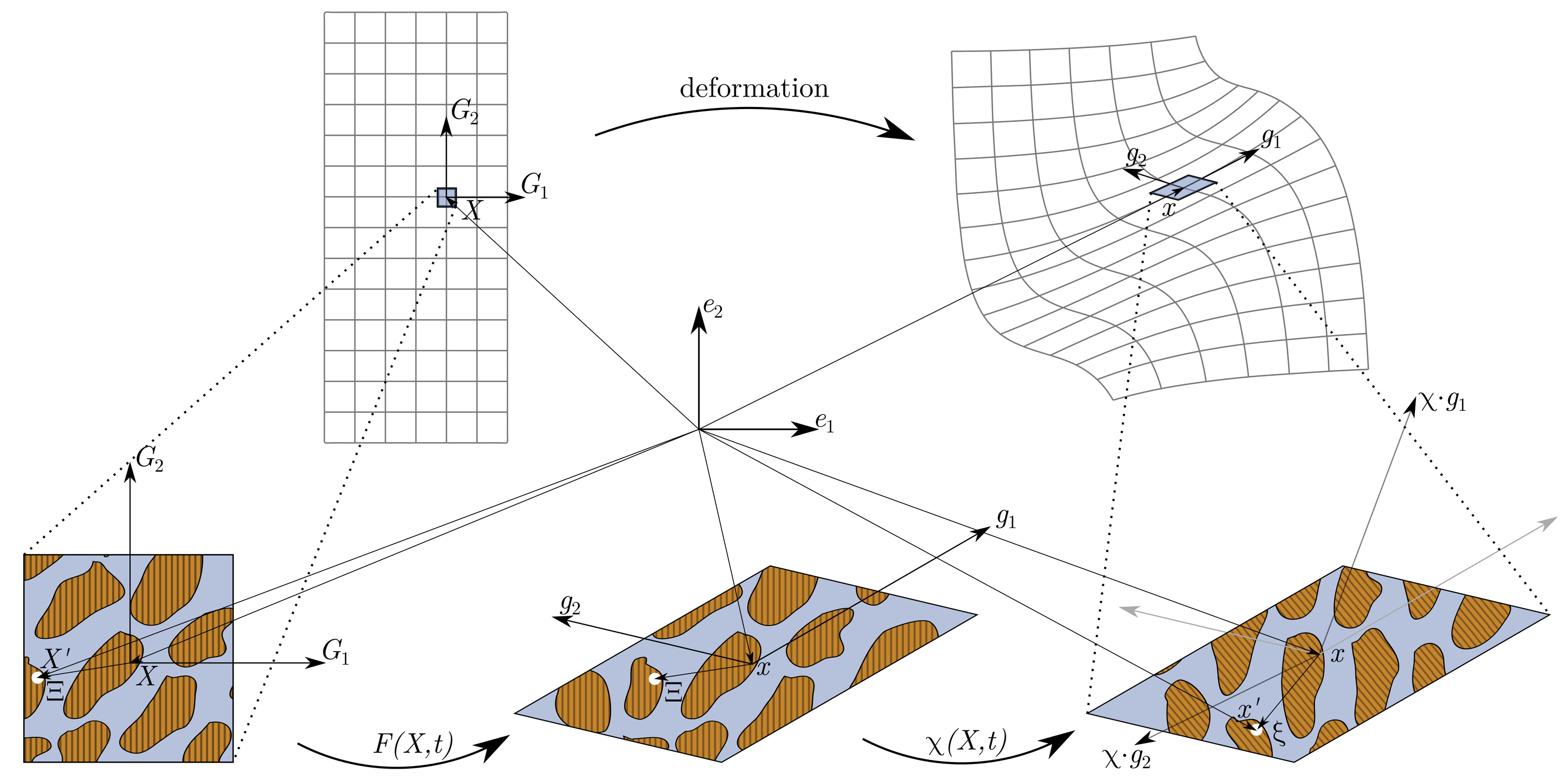

Figure 3: Illustration of the kinematics of the micropolar continuum: The deformation state of a deformable material particle, i.e., a macroelement, is characterized by the macroscopic deformation gradient F, and the microscopic deformation χ, i.e., the microrotation within the macroelement.

The balance equations of the proposed gradient-enhanced micropolar continuum are formed by the linear and the angular momentum equations

$$t^{i}{}_{j,i} + \rho(f_j - \ddot{x}_j) = 0 \quad \text{in } V(t)\,, \tag{1}$$

$$m^{i}{}_{j,i} + \epsilon_{jk}{}^{l}\, t^{k}{}_{l} + \rho(l_j - \sigma_j) = 0 \quad \text{in } V(t) \tag{2}$$

and a second order partial differential equation describing the nonlocal character of damage

$$\tilde{\alpha} - l_{\mathrm{d}}^2 \left(\tilde{\alpha}_{,I}\right)_{,J} G^{IJ} = \alpha_{\mathrm{d}} \quad \text{in } V_0. \tag{3}$$

Example

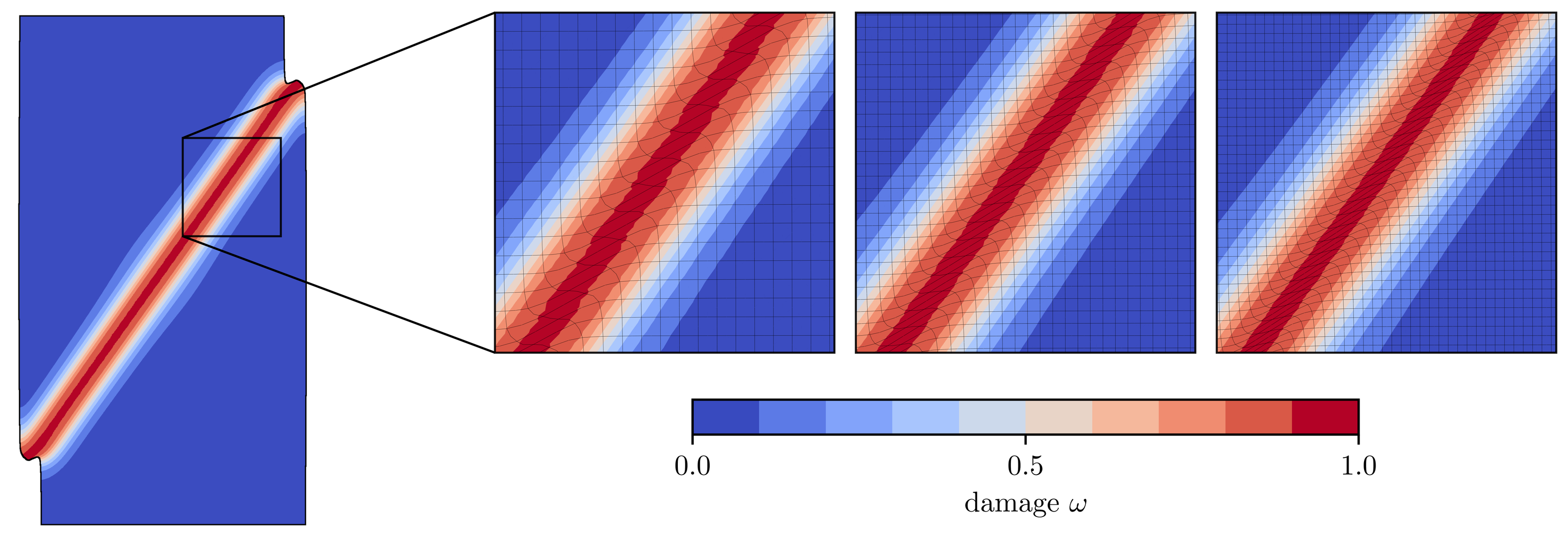

Figure 4: Shear band in a plane strain compression test on a sandstone specimen predicted by the proposed framework for three different finite element meshes. The obtained results are independent of the mesh resolution.

References

[1] M. Neuner, R. A. Regueiro & C. Linder (2022). *Int. J. Solids Struct.* 254-255:111841.

Stanford | ENGINEERING Civil & Environmental Engineering
Stanford University
Stanford | DOERR School of Sustainability