Investigation of a Finite Strain Phase Field Model for Ductile Fracture under Multi-Axial Stress States

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Introduction

Ductile fracture in elastoplastic solids starts at the micro-level where nucleation of small defects concentrated mostly in regions with large plastic deformations, initiates the crack. Micro-voids grow and coalesce into larger voids, and consequently realize their effects at the macro-scale by degrading the stiffness of the body.

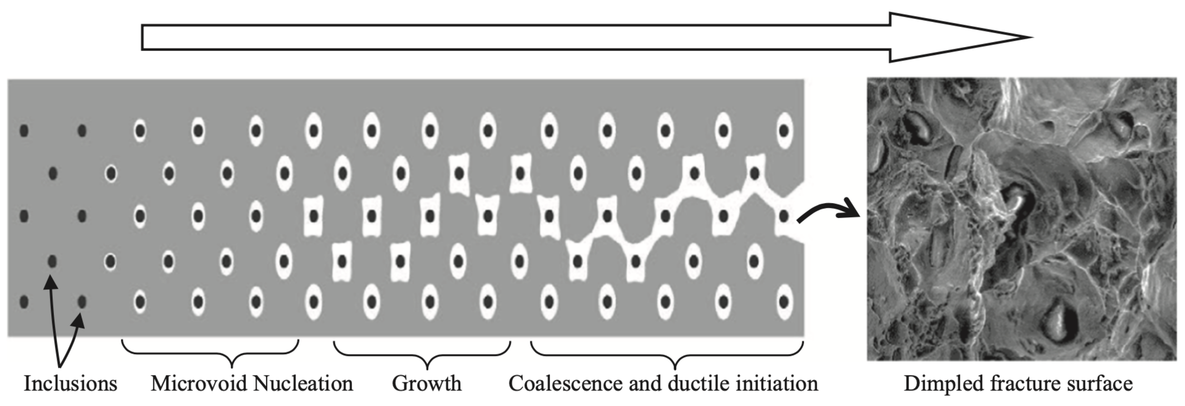

Fig. 1: Micromechanical evolution of a crack in steel (from [1]).

In addition to the plastic deformation, the mechanism of micro-void growth is influenced heavily by the multi-axial stress state experienced by the material. Many studies highlighted the influence of the hydrostatic stress on the ductile behavior of metals through either theoretical studies or experiments [2,3].

Phase field fracture modeling

The phase field approach to fracture, a smeared crack modeling technique, has gained significant attention in the past years. In this approach, the propagation of a crack surface is tracked implicitly through the evolution of an order parameter (the crack phase field d), which transitions smoothly between intact ($d = 0$) and fully cracked ($d = 1$) material points.

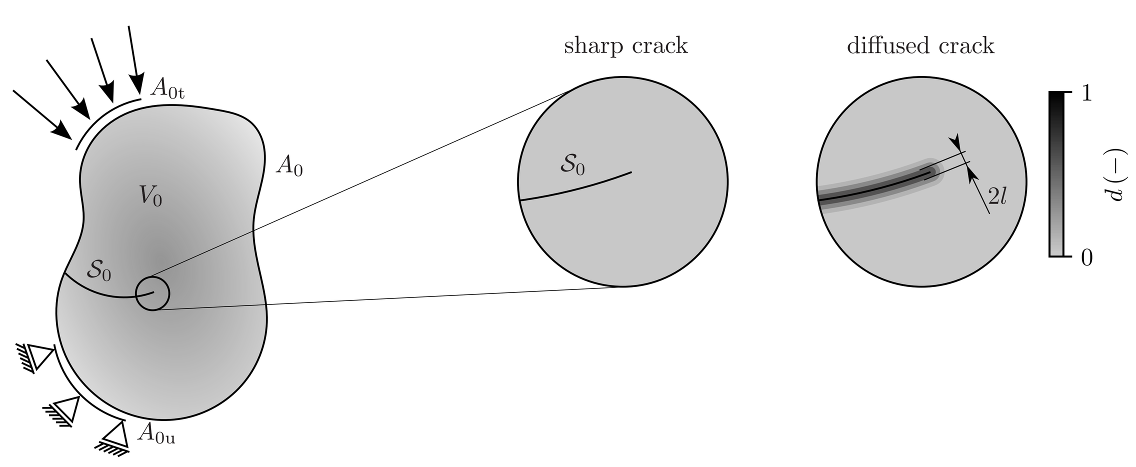

Fig. 2: Crack surface approximation using the phase field parameter d.

In the phase field method, the energy required to create a crack surface is approximated as

$$\int_{\mathcal{S}_0} G_c \, \mathrm{d}\mathcal{S}_0 \approx \int_{V_0} G_c \left(\frac{1}{2l} d^2 + \frac{l}{2} d_{,\boldsymbol{X}} \cdot d_{,\boldsymbol{X}} \right) \mathrm{d}V_0$$

with G_c as the critical fracture energy indicating the resistance of a material to cracking and l as a regularization parameter known as the phase field length scale parameter which controls the width of the diffusive crack. There have been many phase field frameworks in the past decade, however, they mostly focus only on the effects of plastic deformation, and do not consider the range of multi-axial stress states that a body might experience. Therefore, in this work [4], we propose a model that couples the evolution of damage to stress state inside the body.

Fracture under multi-axial stress state

For brittle fracture, G_c is commonly taken as a constant material parameter. On the other hand, in this work for ductile fracture, we assume that G_c degrades depending on the the loading condition and the state of stress and strain experienced by a material point. We propose a fracture toughness degradation function $\mathcal{F}$, governing the evolution of G_c of the form $G_c = \mathcal{F} G_c^0$ with G_c^0 as a material parameter showing the initial crack resistance. We incorporate the Stress-Weighted Ductile Fracture Model (SWDFM) [1], and propose a fracture degradation function of the form

$$\mathcal{F} = (1 - \mathcal{F}_\infty)(1 - D_{\mathrm{SWDFM}})^2 + \mathcal{F}_\infty$$

with $\mathcal{F}_\infty$ as a residual value of fracture toughness. In this equation, the damage quantity D_{SWDFM} is defined as

$$D_{\mathrm{SWDFM}} = \mathcal{C} \int_0^\alpha \left(\left[\exp(1.3T) - \frac{1}{\mathcal{B}} \exp(-1.3T) \right] \exp(\mathcal{K}(|\zeta| - 1)) \right) \mathrm{d}\alpha \, .$$

Therein, parameters $\mathcal{C}$, $\mathcal{B}$, and $\mathcal{K}$ are material constants calibrated based on experimental tests, α is the internal plastic variable, T is the stress triaxiality, and ζ is the Lode angle parameter.

This coupling with the SWDFM has the advantage that through degradation of the fracture toughness, damage concentrates in regions with high values of D_{SWDFM} which takes into account plastic deformation, as well as stress triaxiality and the Lode angle parameter. With that, evolution of damage is related to stress-strain history of the material.

Results

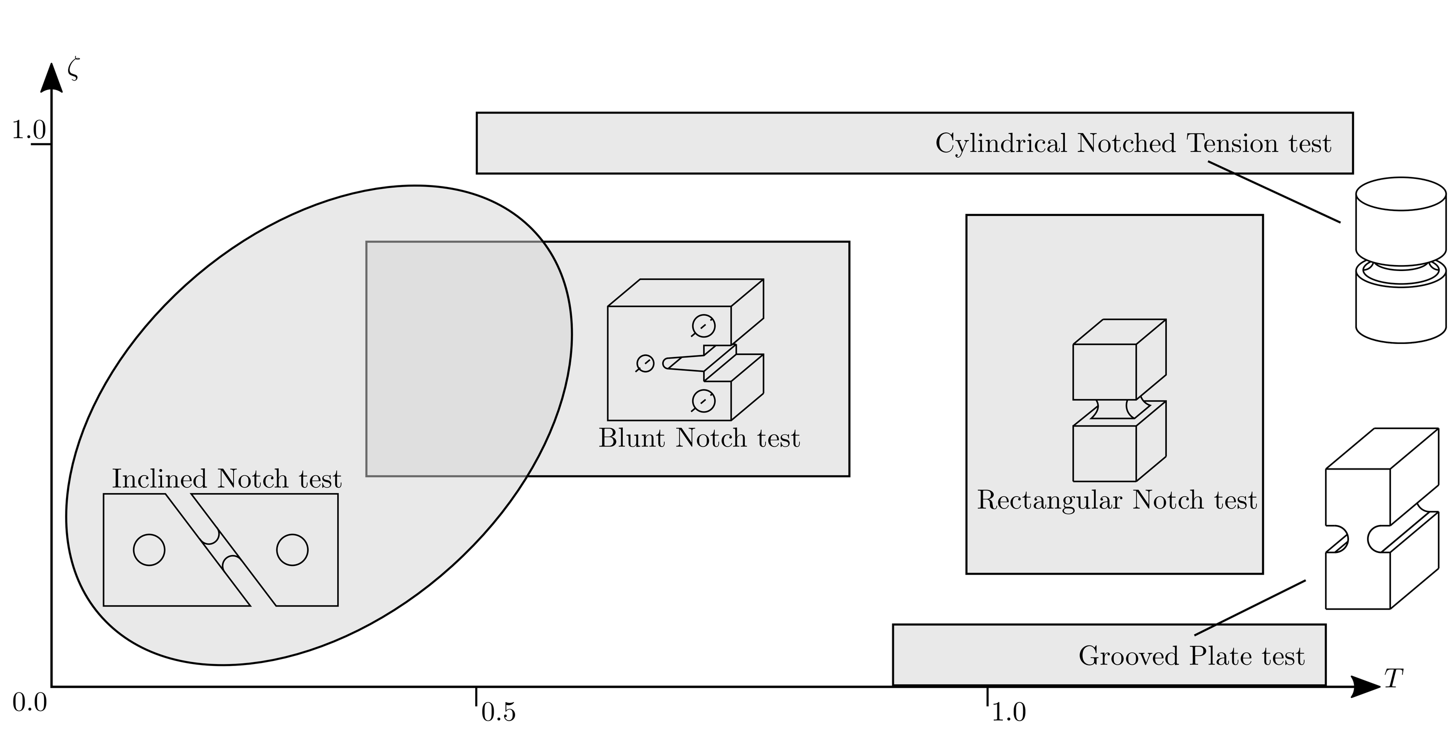

Fig. 3: Level of stress triaxiality and Lode angle parameter at fracture initiation location for various experimental tests (adopted from [1]).

We validate the proposed framework using a numerical study of experimental tests, considering various levels of stress triaxiality and the Lode angle parameter. Fig. 3 schematically presents the levels of stress triaxiality, and values of the Lode angle parameter at the location of crack initiation for different experimental tests. In particular, load-displacement curves for the Cylindrical Notched Tension (CNT) test and Blunt Notch (BN) test are shown in Fig. 4.

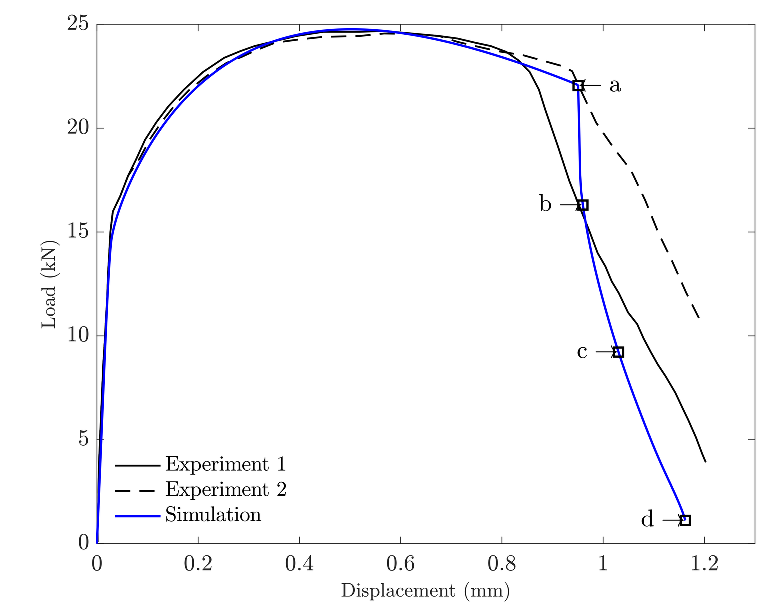

Fig. 4a: Load-displacement curve for the cylindrical notch test.

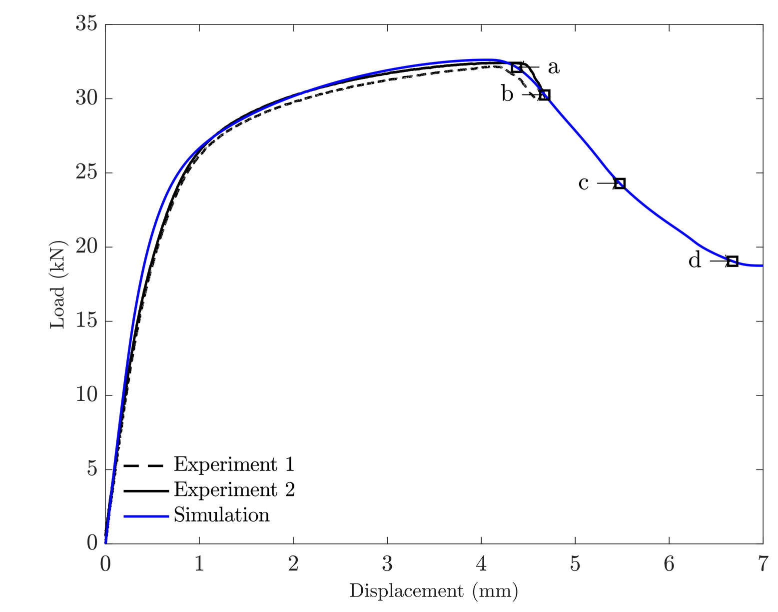

Fig. 4b: Load-displacement curve for the blunt notch test.

Fig. 5 shows selected stages of the evolution of the crack phase field until complete failure for the CNT specimen. For this test, the crack initiates at the center of the specimen, where triaxiality attains its maximum, and then propagates in the radial direction until complete rupture. This test shows the capability of the proposed framework in accounting for effects of the stress triaxiality in predicting both crack initiation location and propagation path.

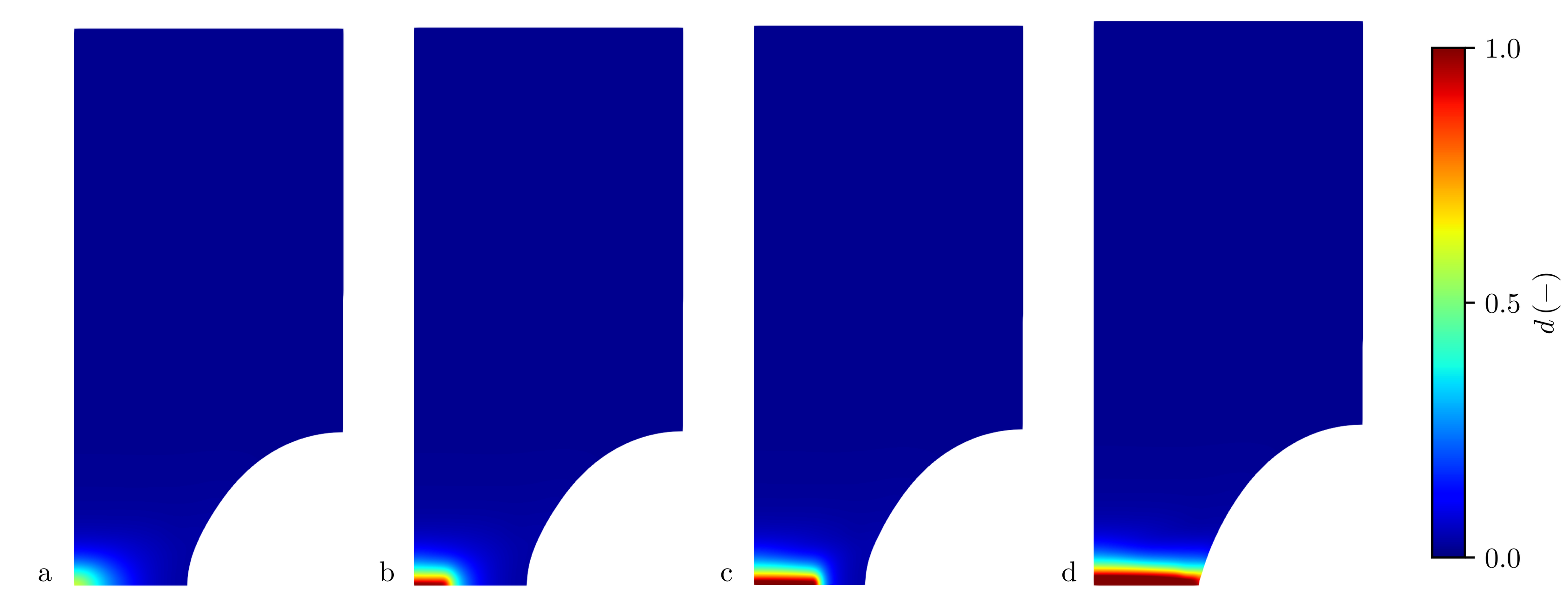

Fig. 5: Evolution of damage contour plots for the CNT test.

The evolution of the phase field parameter for the BN test is visualized in Fig. 6. In this test, the crack initiates at the tip of the notched hole and then propagates horizontally and outward to the surface.

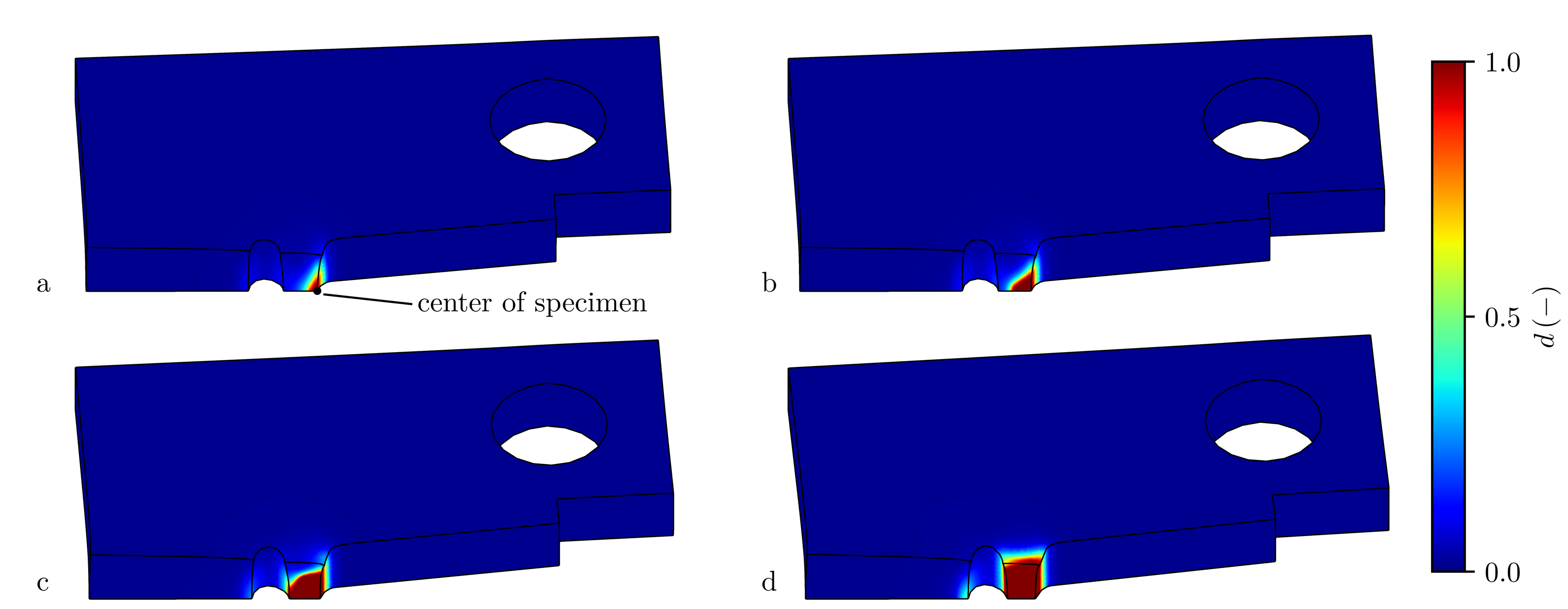

Fig. 6: Evolution of damage contour plots for the BN test.

As shown in Fig. 7, unlike for the CNT example, in the BN test the crack does not initiate at the location of highest stress triaxiality, rather it starts form the location where a combination of the stress triaxiality and plastic deformation leads to crack nucleation. This shows the capability of the SWDFM in accounting for not only the stress state but also the plastic deformation experienced by material points for predicting ductile crack mechanisms.

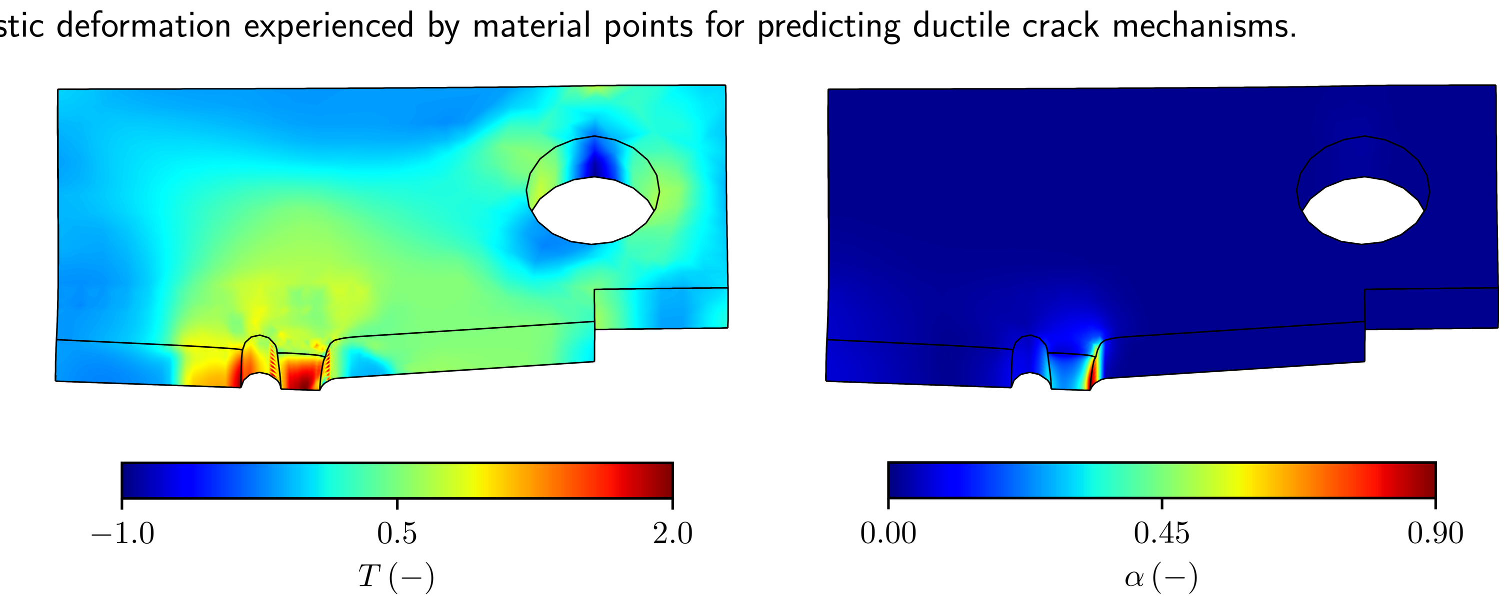

Fig. 7: Distribution of the stress triaxiality T and the internal plastic variable α for the BN test.

Conclusion

The proposed framework assumes a close connection between the stress-strain history of the material and the evolution of damage. To account for various multi-axial stress states, the degradation of the fracture toughness is coupled with the Stress-Weighted Ductile Fracture Model (SWDFM), which allows for accurate prediction of fracture initiation and advancement in elastoplastic solids. The capabilities of the proposed method were validated in a numerical study covering a wide range of stress and strain states.

References

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